Adding & Subtracting Fractions

Before you can add or subtract fractions with UNLIKE denominators, you must first re-write the fractions with a COMMON denominator

Like Denominators

You may combine the numerators ONLY if the denominators are the same !!

If the denominators are the same, then add or subtract the numerators and leave the denominator the same

EXAMPLES:

(addition) $\frac{3}{7} + \frac{5}{7} = \mathbf{\frac{8}{7}}$ (subtraction) $\frac{4}{5} - \frac{3}{5} = \mathbf{\frac{1}{5}}$

Unlike Denominators

If the denominators are not the same, you must first find a common denominator

EXAMPLE: $\frac{2}{3} + \frac{4}{7}$ In this example the common denominator will be 21

$\frac{7}{7} \bullet \frac{2}{3} + \frac{4}{7} \bullet \frac{3}{3}$

$\frac{7 \bullet 2 + 4 \bullet 3}{21} \Rightarrow \frac{14 + 12}{21}$

$= \mathbf{\frac{26}{21}}$

1. Multiply each fraction to change the denominators
2. Rewrite the equation with the common denominator
3. Use the order of operations to solve
4. Simplify

Mixed Numbers

Before you add or subtract mixed numbers, you can change them into improper fractions

EXAMPLE: $1\frac{3}{4} + 2\frac{2}{3}$ In this example the common denominator will be 12

$\frac{7}{4} + \frac{8}{3} \Rightarrow \frac{3 \bullet 7 + 8 \bullet 4}{12}$

$\frac{21 + 32}{12} \Rightarrow \frac{53}{12} = 4\mathbf{\frac{5}{12}}$

1. Write the mixed numbers as improper fractions
2. Determine common denominators
3. Use the order of operations to solve
4. Write answer as a mixed number

Common Conversions

METRIC

The basic prefixes in the metric system, from largest to smallest, are

Kilo Hecto Deka Unit Deci Centi Milli

(Unit represents the base unit such as meter, liter, gram, etc.)

To move from one prefix to another, move the decimal the same number of places in the same direction

Examples: 10 milliliters = 0.01 liters 5 Kilometers = 5000 meters

❖ *If you move to left you're dividing each space by 10, if you move to the right you're multiplying each space by 10*

Metric Mnemonics

King Henry Doesn't Usually Drink Chocolate Milk

VOLUME

U.S.		Metric		U.S. → Metric	
1 gallon	4 quarts	1 Liter	1000 ml	1 qt	946 ml
1 quart	2 pints	1 dl	100 ml	1qt	1.06 L
1 pint	2 cups	1 ml	1cc		

MASS

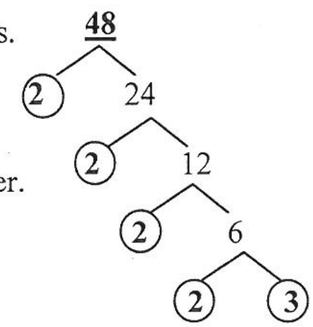
U.S.		Metric		U.S.	Metric
1 lb	16 oz	1 kg	1000 g	1 lb →	454 g
		1 g	1000 mg	2.2 lb	1 kg

LENGTH

U.S.		Metric		U.S.	Metric
1 mi	5280 ft	1 km	1000 m	0.621 mi	1 km
1 yd	3 ft	1 m	100 cm	39.4 in →	1 m
1 ft	12 in	1 cm	10 mm	1 in	2.54 cm

LCM and GCF

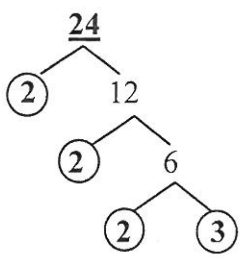
Prime Factorization is to write a composite number as a product of its prime factors. For example, we can use the factor tree to find the prime factorization of 48:



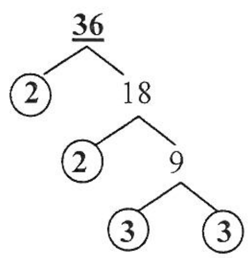
A **Common Multiple** of two or more numbers is any multiple shared by each number. The **Least Common Multiple** is the lowest multiple the numbers have in common.

A **Common Factor** of two or more numbers is any factors that the numbers share. The **Greatest Common Factor** is the largest of these common factors.

Compare the two numbers 24 and 36; start with the factor tree.



$24 = 2 \cdot 2 \cdot 2 \cdot 3$



$36 = 2 \cdot 2 \cdot 3 \cdot 3$

LCM: *Pull out all primes with highest exponent*

24	2	2	2	3	
36	2	2		3	3

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

2^3	3
2^2	3^2

$2^3 \cdot 3^2$

GCF: *Pull out all shared primes with lowest exponent*

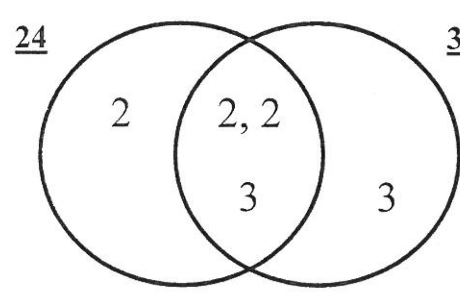
24	2	2	2	3	
36	2	2		3	3

$2 \cdot 2 \cdot 3$

2^3	3
2^2	3^2

$2^2 \cdot 3$

Draw a Venn Diagram –
the shared factors go in the middle



To find **LCM**, *Multiply all* numbers
$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 72$

To find **GCF**, multiply the *Factors shared*
$2 \cdot 2 \cdot 3 = 12$

Multiplying & Dividing Fractions

Multiplying Fractions

1. Multiply the numerators (*across the top*)
2. Multiply the denominators (*across the bottom*)
3. Simplify

EXAMPLE: $\frac{3}{7} \cdot \frac{4}{5}$ $\left(3 \cdot 4 = 12\right)$ $\left(7 \cdot 5 = 35\right)$ $= \mathbf{\frac{12}{35}}$

Simplifying before Multiplying

When a numerator and denominator have common factors, you can simplify before multiplying

EXAMPLE:

$\frac{9}{15} \cdot \frac{5}{9}$ (*cross cancel*) $\frac{\overset{1}{\cancel{9}}}{\underset{3}{\cancel{15}}} \cdot \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{9}}}$

one 9 goes into 9 **one** time
one 5 goes into 15 **three** times

$\frac{1}{3} \cdot \frac{1}{1} = \mathbf{\frac{1}{3}}$

Dividing Fractions

1. Leave the first number alone
2. Change divide (÷) to multiply (•)
3. Find the reciprocal (flip) of the second number
4. Divide common factors (simplify)
5. Follow the procedure for multiplying fractions

EXAMPLE:

$\frac{2}{9} \div \frac{2}{5} \Rightarrow \frac{2}{9} \cdot \frac{5}{2}$

$\frac{\overset{1}{\cancel{2}}}{9} \cdot \frac{5}{\underset{1}{\cancel{2}}} \Rightarrow \frac{1}{9} \cdot \frac{5}{1} = \mathbf{\frac{5}{9}}$

Dividing with Mixed Numbers

First change all mixed numbers into improper fractions, then follow the rules for dividing fractions

EXAMPLE:

$1\frac{3}{4} \div 2\frac{5}{8} \Rightarrow \frac{7}{4} \div \frac{21}{8} \Rightarrow \frac{7}{4} \cdot \frac{8}{21} \Rightarrow \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{4}}} \cdot \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{21}}} \Rightarrow \frac{1}{1} \cdot \frac{2}{3} = \mathbf{\frac{2}{3}}$

Order of Operations

1. Do the operations in grouping symbols, starting with the innermost and moving outward.

   Grouping symbols include ( ) [ ] { }

2. Calculate exponents.

3. Perform all multiplications and divisions working from **left to right**.

4. Perform al additions and subtractions working from **left to right**.

Here is an acronym to help remember the order of operations: **P E M D A S**

P	**P**arenthesis
E	**E**xponents
M D	**M**ultiplication & **D**ivision *(worked left to right)*
A S	**A**ddition & **S**ubtraction *(worked left to right)*

Or, remember this sentence: **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally

Properties of Addition & Multiplication

Identity Properties

- Identity Property of Addition
  - When zero is added to any number, the resulting sum is that number.
    $5 + 0 = 5$ $\quad$ $0 + 2 = 2$
- Identity Property of Multiplication
  - When one is multiplied by any number, the resulting product is that number.
    $5 \cdot 1 = 5$ $\quad$ $1 \cdot 2 = 2$

Associative Properties

- Associative Property of Addition
  - When adding three or more numbers, changing the grouping does not change the sum. *Associate means to join or group*
    $(3 + 4) + 5$ $\quad$ $3 + (4 + 5)$
    $7 + 5 = 12$ $\quad$ $3 + 9 = 12$
- Associative Property of Multiplication
  - When multiplying any three numbers, changing the grouping does not change the product. (*Associate = group*)
    $(3 \cdot 4)5$ $\quad$ $3(4 \cdot 5)$
    $12 \cdot 5 = 60$ $\quad$ $3 \cdot 20 = 60$

Commutative Properties

- Commutative Property of Addition
  - When adding any two or more numbers, changing the order does not change the sum. *Commute means travel or move*
    $3 + 4 = 4 + 3$
    $7 = 7$
- Commutative Property of Multiplication
  - When multiplying any two or more numbers, changing the order does not change the product. (*Commute = move*)
    $3 \cdot 4 = 4 \cdot 3$
    $12 = 12$

Inverse Property

- Inverse Property of Addition
  - When a number and its additive inverse are added to one another, the sum is 0.
    Additive inverses are called opposites. (change sign)
    $6 + (-6) = 0$ $\quad$ $-12 + 12 = 0$
- Inverse Property of Multiplication
  - When a number and its multiplicative inverse are multiplied, the product is 1.
    Multiplicative inverses are called reciprocals. (flip it)
    $7 \cdot \frac{1}{7} = \frac{1}{7} \cdot 7 = 1$

Distributive Property

- Distributive Property of Multiplication over Addition
  - The product of a number (multiplier) and a sum can be found by multiplying the number by each addend (both numbers in the sum) "*Multiplication distributes over additions*"

$$\begin{aligned} 3(10 + 2) &= 3 \cdot 10 + 3 \cdot 2 \\ &= 30 + 6 \\ &= 36 \end{aligned}$$

Proportions

A proportion is a name we give to a statement that two ratios are equal.
**remember a ratio is quotient of two quantities, AKA fraction!*

$$\frac{a}{b} = \frac{c}{d}$$

When two ratios are equal, then the cross products of the ratios are equal.

$$a \bullet d = b \bullet c$$

Here are some example proportions that use words to define the location of information:

a) When setting up a proportion for percentage, make sure you use the "part" over "whole" rule.

$$\frac{\text{part}}{\text{whole}} = \frac{x}{100} \qquad \frac{\text{increase/decrease}}{\text{original}} = \frac{x}{100} \qquad \frac{\text{change}}{\text{original}} = \frac{x}{100}$$

b) When setting up your ratios for a proportion, remember that the relationship of units must be the same for each ratio. (*tops should match, & bottoms should match OR left side should match, & right side should match*)

$$\frac{\text{miles}}{\text{hour}} = \frac{\text{miles}}{\text{hour}} \qquad \frac{\text{dollars}}{\text{gallon}} = \frac{\text{dollars}}{\text{gallon}} \qquad \frac{\text{inches}}{\text{feet}} = \frac{\text{inches}}{\text{feet}}$$

OR

$$\frac{\text{miles}}{\text{miles}} = \frac{\text{hours}}{\text{hours}} \qquad \frac{\text{dollars}}{\text{dollars}} = \frac{\text{gallons}}{\text{gallons}} \qquad \frac{\text{inches}}{\text{inches}} = \frac{\text{feet}}{\text{feet}}$$

Example Problem 1:

Yesterday, a cup of coffee was \$1.75, today a cup of coffee cost \$1.82. What was the percent increase of the cost of a cup coffee?

1) First, choose a percent proportion (*above*) that will work with this problem: $\frac{\text{increase}}{\text{original}} = \frac{x}{100}$
2) Next, fill in the information from the word problem

   What is the increase? *\$1.82 – \$1.75 = \$0.07*
   What was the original price? *\$1.75*

$$\frac{0.07}{1.75} = \frac{x}{100}$$

3) Cross multiply

$$1.75 \bullet x = 0.07 \bullet 100$$
$$1.75x = 7$$

4) Solve for the variable "x"

$$\frac{1.75x}{1.75} = \frac{7}{1.75}$$
$$\mathbf{x = 4}$$

Example Problem 2:

A recipe calls for 3 cups of sugar for 8 servings, how many cups of sugar are needed for 12 servings?

1) Use the ratio of cups to servings
2) Cross multiply
3) Solve for "x"
4) Simplify

$$\frac{3}{8} = \frac{x}{12} \Rightarrow 8x = 36 \Rightarrow \frac{8x}{8} = \frac{36}{8}$$

$$x = 4\tfrac{4}{8} \Rightarrow \mathbf{4\tfrac{1}{2}\ cups}$$

Rational and Irrational Numbers

Rational

Rational Numbers can be expressed as the quotient of two integers (*i.e. a ratio or fraction*) with a denominator that is not zero.

Examples of Rational Numbers:

- 5 You can express 5 as $\frac{5}{1}$ which is the quotient of the integer 5 and 1.
- $\frac{2}{4}$, $\frac{0}{4}$, basically any fraction because fractions are quotients of two integers
- $\sqrt{9}$ is rational because you can simplify the square root to 3 which is the quotient of the integer 3 and 1
- $.\overline{11}$ All repeating decimals are rational.

  1) let x = .1

  2) 10x = 1.1

  3) 10x -1x = 1.1 -.1 (*i.e. subtract top equation from bottom equation*)

  2) 9x = 1

  3) x = 1/9 . Yes, the repeating decimal .1 is equivalent to the fraction 1/9

- .9 is a rational number because it can be expressed as $\frac{9}{10}$ (*All terminating decimals are rational numbers*)
- .73 is a rational number because it can be expressed as $\frac{73}{100}$

Irrational

Irrational Numbers are decimal numbers that don't end (*non-terminating*) and never repeat. They cannot be expressed as the quotient of two integers (*i.e. a ratio or fraction*) such that the denominator is not zero

Examples of Irrational Numbers

- $\sqrt{7}$ You cannot simply this square root (like we did with $\sqrt{9}$) so this number is irrational.
  ALL irreducible square roots are irrational
- $\frac{5}{0}$ If a fraction has a denominator of zero, it is irrational
- π Pi is probably the most well-known irrational number out there, it is a non-terminating decimal

Signed Integers

Addition

Do the numbers have the ***SAME*** SIGN?

YES - *Same* Signs = find the *sum*:

$(-3) + (-6) = -9$

$(+4) + (+5) = (+9)$

NO - *Different* Signs = find the *difference*:

$(+5) + (-7) = (-2)$

$(-4) + (+6) = (+2)$

❖ **Either way, keep the sign of the *LARGER* number**

Subtraction

First, change the SUBTRACTION problem to an ADDITION problem;
Then, follow the rules for solving ADDITION problems (*above*)

$(-6) - (+2) =$

When changing subtraction to addition, you must rewrite the problem:

1) The first number stays the same: (-6)
2) Change the operation: $(-6) +$
3) Switch the NEXT SIGN: $(-6) + (-2)$
4) Follow the rules for addition: $(-6) + (-2) = (-8)$

❖ **Subtract means "*Add the Opposite*"**

Subtract means:	$(+4) - (-5) =$
Add the opposite	$(+4) + (+5) = (+9)$
Subtract means:	$(-7) - (-3) =$
Add the opposite	$(-7) + (+3) = (-4)$
Subtract means:	$(+4) - (+9) =$
Add the opposite	$(+4) + (-9) = (-5)$

Multiply or Divide

Same signs = positive

$+ \cdot + = +$ $\quad - \cdot - = +$

$+ \div + = +$ $\quad - \div - = +$

Different signs = negative

$- \cdot + = -$ $\quad + \cdot - = -$

$- \div + = -$ $\quad + \div - = -$

Solve an Equation for "x"

The main thing to remember when solving an equation for a variable is that the equal sign (=) indicates that both sides (Left Hand Side – LHS, & Right Hand Side – RHS) are equal to each other; therefore when solving, whatever you do to one side of the equation, you **MUST** do to the other side in order to keep the equation balanced, or equal, on both sides.

Addition and subtraction are inverse operations; they "undo" each other (i.e. $10 + 9 - 9 = 10$).
To solve equations using addition and subtraction, first decide which operation has been applied, then use the inverse operation to undo this.

Examples:

$$\begin{array}{rcl} x + 7 & = & 15 \\ -7 & & -7 \\ \hline x + 0 & = & 8 \\ \mathbf{x} & \mathbf{=} & \mathbf{8} \end{array} \qquad \begin{array}{rcl} x - 17 & = & 9 \\ +17 & & +17 \\ \hline x + 0 & = & 26 \\ \mathbf{x} & \mathbf{=} & \mathbf{26} \end{array} \qquad \begin{array}{rcl} 12 + x & = & 16 \\ -12 & & -12 \\ \hline 0 + x & = & 4 \\ \mathbf{x} & \mathbf{=} & \mathbf{4} \end{array}$$

Multiplication and division are inverse operations; they "undo" each other (i.e. $5 \cdot 2 \div 2 = 5$).
To solve equations using multiplication and division, first decide which operation has been applied, then use the inverse operation to undo this.

Examples:

$$\begin{array}{rcl} 2x + 3 & = & 21 \\ -3 & & -3 \\ \hline \frac{2x}{2} + 0 & = & \frac{18}{2} \\ \hline x + 0 & = & 9 \\ \mathbf{x} & \mathbf{=} & \mathbf{9} \end{array} \qquad \begin{array}{rcl} -5x - 8 & = & 32 \\ +8 & & +8 \\ \hline \frac{-5x}{-5} + 0 & = & \frac{40}{-5} \\ \hline x + 0 & = & -8 \\ \mathbf{x} & \mathbf{=} & \mathbf{-8} \end{array} \qquad \begin{array}{rcl} 9 + 7x & = & 58 \\ -9 & & -9 \\ \hline 0 + \frac{7x}{7} & = & \frac{49}{7} \\ \hline 0 + x & = & 7 \\ \mathbf{x} & \mathbf{=} & \mathbf{7} \end{array}$$

Translating English Phrases into Algebraic Expressions

	ENGLISH PHRASE	ALGEBRAIC PHRASE
1.	Twice as much as a number	$2x$
2.	Sixteen <u>**less**</u> twice a number	$16 - 2x$
3.	Sixteen <u>**less than**</u> twice a number	$2x - 16$
4.	Five more than a number	$x + 5$ OR $5 + x$
5.	Three more than twice a number	$3 + 2x$ OR $2x + 3$
6.	A number decreased by seven	$x - 7$
7.	Ten decreased by a number	$10 - x$
8.	<u>**There are six more x than y**</u>	$x = y + 6$ OR $x - 6 = y$
9.	Shari's age (x) four years from now	$x + 4$
10.	Lyla's age (x) ten years ago	$x - 10$
11.	Number of cents (dollars) in x quarters	$25x$; (\$0.25x)
12.	Number of cents (dollars) in (15 – x) dimes	$10(15 - x)$; [\$.10(15 – x)]
13.	Number of cents (dollars) in 2x nickels	$5(2x)$; [\$0.05(2x)]
14.	Separate seventeen into two parts	x and $(17 - x)$
15.	Invest $1000 in two different accounts	x and $(1{,}000 - x)$
16.	Two consecutive integers	x and $(x + 1)$
17.	Two consecutive even integers	x and $(x + 2)$
18.	Three consecutive odd integers	x and $(x + 2)$ and $(x + 4)$
19.	Distance traveled in three hours at x mph	$3x$
20.	Sum of a number and eight	$x + 8$ OR $8 + x$
21.	Difference of six and a number	$6 - x$ OR $-(6 + x)$
22.	Product of a number and its reciprocal	$x(1/x)$
23.	Quotient of a number and twenty-two	$x/22$
24.	Three is four more than a number	$3 = 4 + x$

<u>**NOTES:**</u>

"**Twice as much**" means "multiply by 2"

"**More than**" means "add" and "**increased by**" means "add"

"**Decreased by**" means "subtract"

"**Percent of**" means "multiply"

"**Is**", "**was**", "**will be**", "**is equal to**", become the equal sign (=) of an equation in algebra