**Design & Analysis of Algorithms (Questions 5 – 8)**

**Problem 5**

For the rabbits in a certain zoo, the population increases in such a way that:
$$T(N) = 2\ T(N-1) + 3$$
$$T(1) = 4$$
where *T(N)* is the population of the *N*-th year. Unfold this recurrence relation
and obtain an expression for the number of rabbits in the zoo in the *N*-th year.
At one stage in Australia, the number of predators for rabbits was very low
and rabbits were more or less multiplying at will. Can you imagine the plight
of the people there if the above recurrence were obeyed? (Of course, it did not
hold true, but Australia still had a problem of too many rabbits.)

**Problem 6**

AB-CD, a company that sells audio CDs, wants to offer for free to its customers the latest
CD of MammaDonna, a hit singer of the 90's. It wants, however, to give only one CD per
household. Members of the same household are defined as those that share the same
address. AB-CD keeps the records of its *N* customers in *an unsorted linked list*, where
each element contains a name and an address. Thus, AB-CD decides to prune the
customer list such that only one customer per address is kept, i.e., every household
appears only once. Describe an efficient algorithm, in pseudocode or English, to prune
AB-CD's customer list. Make sure that you are doing better than the naive, inefficient,
*$O(N^2)$* algorithm that compares every possible pair of records. You may use any
additional data structure.

**Problem 7**



*A "true" medieval story...*
Sir Arthur de Templar was paid 27 gold coins for the information he provided on the last
known location of the Holly Grail. Soon, from the highly trusted sources, he has learned
that one of the 27 gold coins is not fully gold (it has some admixture of iron and copper
and hence its weight differs (<u>either heavier or lighter</u>) from the weight of the other 26 true
gold coins). Sir Arthur paid a visit to his closest friend drug-maker Mr. Drugless and in a
few seconds they identified the fake coin. Legend says that they used just an old
weighing device of Mr. Drugless (depicted on the Figure above) and could identify the
fake coin in only 4 weighings.
>    1. Repeat the procedure used by Sir Arthur de Templar and Mr. Drugless for
>    identifying the fake coin among 27 gold coins in just 4 weighings.
>    2. Extend it to an algorithm for identifying a single fake coin among *N* given gold
>    coins using minimum number of weighings. How many weighings do you need?
(A weighing is an operation of putting *k* coins on one plate of the device and *k* other coins
on the other plate (*k=1 or 2 or 3 ...*) and checking if the device is in equilibrium, and if
not, which plate is heavier, lighter.)

## Problem 8

An Euler tour of an un-directed graph $G$ with $n$ vertices and $m$ edges is a cycle that traverses each edge of $G$ exactly once. Such a tour always exists if the degree of each vertex in $G$ is even. Describe an $O(n+m)$ time algorithm for finding an Euler tour of such a graph $G$. Analyze its running time.