Hello, class. Welcome to our Time Value of Money analysis tutorial. And we're going to start off. We have a number of tabs here that we're going to go through today. We're going to do future value of lump sum, present value of a lump sum, an ordinary annuity, present value of uneven cash flows. We're going to work on the rate formula and then finish up with an investment calculator that I'm going to show you how to go ahead to put together.

So let's get started. The first tab that we're going to be working on is what we're looking at here. It's the future value of a lump sum. And for this particular problem, we're going to calculate it in three different ways. And I think there's some value in doing that so you kind see exactly what's happening through this compounding exercise. And then clearly, one of the ways is the preferred, and basically it's a demonstration of how powerful Excel is and the functions that are behind it.

So let's go ahead and get started. For this particular problem, what we're looking at is the future value of a $25,000 investment. What's this $25,000 investment going to be worth in five years from now at a 10% return rate?

So out here in year five, we're looking at what's this worth? And there's a compounding effect that's happening here. We're going to demonstrate how to go ahead and solve for this problem. So you're familiar with this calculation because we have used it in a couple of the previous tutorials.

And it goes it goes like this. So we're going to take our initial investment. We're multiplying it times one plus the interest rate here. And when we do that, we find out that our $25,000 investment in year one, at the end of year one, is going to be worth $27,500.

When we factor year two, we use the base now from year one. This is how compounding works. And we apply that same 10% increase again. And at the end of year two, our investment's worth $30,250. To And we're going to calculate the remaining years here as well.

And year four. And finally, year five. OK. So it's kind of a tedious exercise and method for calculating this, so we're going to work on a couple other ways to do it, preferred ways. But before we do that, let's calculate the amount of interest that has been fetched from this investment. So it's just equal to $27,500 minus $25,000, or in the first year we generated $2,500 dollars in interest earned. And we can pull this calculation across, and we find out that we had a total earnings and interest of $15,262.

And our year five value of this investment, which we answered this question, is $40,262.75. Initial investment of

$25,000 over five years at a 10% interest rate compounded each year.

So let's write the manual formula for this now, so methods for solving future value. This is going to be the manual formula. We're going to basically take everything that we did here and condense this down into one formula. The power of this is if you had many, many periods of information that you were working with here, you would not want to go through and have to calculate this year by year.

But what we want is we want to know what is this at the end of year five? And we have a formula that we can use to do that. So again, the present value comes in as a negative number. It's a cash outflow. What comes back to us is a positive, which is the $40,262. So that's why it's entered as a negative number to start with.

We wouldn't have to do this if we were just calculating it manually, but because we're going to use the Excel formula, we have to enter it as a negative. So that's why we started off with it like this. So it's the present value times 1 plus the interest rate and then a parentheses around that. We're going to put a parentheses in front and we're going to take this to the power of the number of periods, that we have, which is five.

And again, this symbol here, this P hat symbol is right above the 6 on your keyboard if you're working along. Now we need to now multiply this times negative 1 to bring us back to a positive number. And when we do that, we see that we're able to write this, and it equals the same value that we got above here doing this year by year, or the $40,262.75.

There's an easier way to do this, and the easier way to do it is by using Excel function keys. And if we go to Financials under Function, we go down to Future Value. We end up with this box out here, and it asks for all of the different variables that we have to work with here. So the rate in this calculation is 10%. The number of periods is five. There are no payments in this exercise here. But there is a present value, and that present value is $25,000. And when we call that up, we get the exact same amount.

And by far the easier of the two of the ways to calculate this. This formula gets messy. You're going to see how messy it gets when we move over here to this compounded quarterly number that many, many things going on. So an easy way to calculate this is using the Financial functions in Excel to work this problem for us.

So to work this quarterly, the manual formula starts off the same way, the present value times 1 plus the interest rate. Now because it's compounded quarterly, we have to divide this into four equal units. So we divide the interest rate by four. We've got to place our parentheses correctly to capture that. And then again, we're going to take this to the power of the number of periods that are in place, which is five. But in this case, it's five times four because we've broken this down into four equal yearly periods in five years. So it's times 4, the number of years times 4.

And then we're going to multiply all of this times negative 1. And what we get is 40,965. So you can see, going from compounded yearly at 40,262, if we compound this quarterly over the five-year period, we get a return of 40,965. So we gained a little additional in interest just by the additional compounding on the quarterly basis. Now that easier way to do this again is using the future value in Excel. And the rate for this, again, is 10% divided by 4, or 2.5% per quarter of the year. The number of periods is the five years times 4. And then our present value, again, is this 25,000.

And when we call that up, we see that we get the exact same number that we did with our manual calculation. Now you've probably heard this term before, the effective interest rate or effective rate of interest. If we want to find the effective rate of interest for a yearly amount that's compounded quarterly, we can calculate that. And the formula is like this. It starts off with an equal and a 1. And it's 1 plus our 10% divided by that quarterly amount, or the 4. And we need to put parentheses around all of this, capture it.

And then we're going to take that. We're going to raise that to the fourth power. And when we do that, we get 110.38%. Now that's not quite right. We have one more step to do in this. Because we added 1, which we had to do to calculate this-- we had to have this 1 plus the fraction that we're raising to the power-- we have to now subtract that 1 back out of this formula. So we make sure we put our parentheses around this, and we subtract it by 1. And we find that the effective rate in this calculation is 10.38%. So the actual rate is 10. But if we're using the quarterly compounding, we're actually getting a return of 10.38.

Now we can test that formula. And the way that we do that is if we go back to our future value, and we're saying that this problem that we have here, we know the quarterly compounded return is 40,965. If we drop this in and don't compound this and take it over five years, and take this present value without doing the compounding, because we've already calculated it within our effective rate, we can test that formula. And we find that it works out to the exact same as what the compounded rate would be.

So it's a test to make sure that our effective rate is actually what we calculated it to be. So 10%, or 10.38%, if compounded quarterly, and it uses the same amount.

So that completes the section of the future value of a lump sum. We're going to move on to the present value of a lump sum calculation. And now, if you recall in the future value of a lump sum, we're moving along that time spectrum from left to right. Now we're going to move from right to left. And that means that we know what the future value is. We want to know what the present value is. And in words, one way to say that is-- and it says it in this problem here-- is in three years you'll receive $18,000. What is the value in today's dollars at a 12% rate of return?

Well, we know that if we're going to receive this in three years, we know in today's dollars it's not going to be worth as much just because of that time value and that compounding, or in this case, it's called discounting when we go from right to left across this time spectrum. And the calculations are similar, but there is a difference here.

So we're going to demonstrate this. We're going to do it this kind of manual way to start with, or this long version. Then we'll do a manual formula, and then we'll get into the-- again, the preferred way is through Excel, and knowing in Excel where to find these formulas.

So the first step here is at the end of year two, this is at the end of year three what this would be worth. At the end of year two, this investment would be worth $18,000. And it's divided by 1 plus our yearly interest rate. So that's the difference. We're dividing this now, dividing that amount when we go from right to left. And at the end of year two, that investment would be worth $16,071.

And at the end of year one, we take this base of year two now, and divide that by 1 plus our yearly interest rate. At the end of year one it was worth $14,349. And at the beginning of our investment or beginning of year one, it is worth 1 plus. Again, that's 12% dividing, going from right to left. So in today's dollars this investment's worth $12,812.

This is critical. And this is used a lot in bond valuations, this type of calculation here and using these present values.

So let's move on now and let's attempt to write the manual formula for this, and that's combining all of this into one formula. And that's going from 18,000 and determining what the present value of that is. So the calculation for that is our future value divided by 1 plus interest rate. And then the P hat symbol over a three-year period. And we'll bracket this or put our parentheses around it, and then multiply all of this times negative 1, again, to account for our future value at the negative amount.

And when we do that, we see that we're able to bring all of that into this one condensed formula, which takes us from that $18,000 investment to the present value of $12,008.12. Let's use the financial functions in Excel. And we cruise down here, and we're going to pull this present value, PV, present value. Again, the box looks similar to what we were using in the future value. It calls for the rate, calls for the number of periods in this investment. And then the future value is this 18,000.

And when we pull that up, we see we get the exact same $12,812. So you can see. Most people like, once you start to learn these financial functions, and all of the functions, really, in Excel and start relying on those, there's a lot of power behind that. Again, it's just knowing which ones to use in which situations. And part of that just comes with practice and seeing different problems and working through different problems to determine which formulas and which functions you're going to want to use to solve each problem.

Now let's go on to the compounded quarterly for this present value problem. And this one gets rather messy. So again, we start with the future value. We're going to divide this by 1 plus this 12%. And that's again divided by 4 because we're doing this on a quarterly basis. And we're going to set our parentheses up correctly. Right now it's balanced. We got parentheses going both ways. We got two going to the right and two back to the left. So we're OK. We're going to raise this to, again, the three years, and then times 4 because we're compounding it quarterly. And one set of brackets around all of this times negative 1.

And we get an output of 12,625. Because it's being compounded quarterly, we find that that investment is worth less than if it was when we're discounting this at the yearly rate. Now let's go to our financial functions keys again and pull up the present value again. Again, the rate here is the 12% divided by the four periods because of the compounded quarterly. The number of periods are the three years times the quarterly amount, or the four, and then the future value of 18,000.

And when we do that, we get the exact same number that we did in this rather messy formula that it's easy to make a mistake on this. I think there's some value knowing, though, how to write these. Again, many of you are going to be in positions where you end up having to write your own formulas to solve problems out there. They're unstructured. There's no necessarily a workbook or anything to show you. You end up having to think through problems and write your own formulas to do that. And this is good practice. It's good to know exactly how this works and kind of the sequence of how all of this fits together as well.

So that's the present value of a lump sum calculated three different ways. And let's move on to an ordinary annuity. So in this case here, in the first two problems, we were working with lump sum payments. In this case here, we're going to be working with an annuity or payments, three equal $15,000 payments over a three-year period. So for each period, there's going to be a $15,000 installment that's made, not one lump sum of 15, but $15,000 invested over a three-year period.

And in this case we don't write the manual formula. We're only going to do Excel because the formula gets really messy. So let's go through this and solve this. And we're going to solve this two different ways. And solve an ordinary annuity, which is this first line. And then we're going to solve what we call an annuity due, and I'll explain that when we get to that piece of it.

But again, we're back to our financial functions. This is a future value. Again, the rate that we're going to calculate is 8% in this case. The number of periods here are three. And in this case we've been using the present value. Now we're going to use the payment, which is for the annuity. So we drop this $15,000 under the payment amount. And when we do that-- I'm in the wrong bucket here. Let's move up here to this one.

So we're going to call up the rate, number of periods, three, and again, the payment is $15,000. And we drop that in and we get an output of $48,696. So these three equal $15,000 payments, at the end of three years, are actually worth $48,696. And that's because of the interest that's compounding. Of course, the interest on that last payment is not as valuable as the payment that came in for that first year. And it's all calculated within this problem. And Excel does that nicely for us.

Now we can do this also on a quarterly basis. And it's similar to the problems that we've been working on. It's the rate divided by 4 because it's quarterly. The number of periods, again, are three years times 4. And you have to-- in this case, we pull up that $15,000, but in this case we have to divide that by 4. So now it's going to be $15,000 divided by 4 over those four quarterly payments per year.

And when we do that, we find that it makes a difference. It's now $50,295 is what that's worth, these three equal $15,000 payments. And it's the compounding interest at work for us here, and even more at work under this quarterly piece.

So this next annuity, we're going to use these same conditions, but it's factored a little bit differently. The first part of this problem is the same. The rate is 8%. The number of periods are three. It's also $15,000 equal payments. But down here under type, it says as a value representing the timing of the payment. Payment at the beginning of the period equals 1.

An annuity due is a payment that is made at the beginning of the period or at the beginning of the year. If you don't put anything in here, it's assumed that the payment comes at the end of the year, which is what we calculated up here for this what we call an ordinary annuity.

But annuity due is the payment is made at the beginning of the period or at the beginning of the year. And to make that happen you just drop a 1 in here. And we find out that it does make a difference. You have those dollars working for you for that extra period. And makes a pretty significant difference.

People talk about that, too, like on your house payment and stuff. If you're able to make a house payment at the beginning of the period as opposed to the end, over the length of the loan you will see a significant difference in the amount of interest that you pay.

In this case here, doing this on a quarterly basis, we go back to future value again. And our rate is 8 divided by 4. And our number of periods are the number of years times 4. Again, payment is $15,000 divided by 4. And then we're going to do this as an annuity due with the payments at the beginning, now, of the period.

What's interesting here is this looks like an anomaly. But this is correct. And in this case here, under this quarterly amount, you can see that there is a difference here. And I'm not going to tell you right now exactly why that is. I want you to think about that, why this amount, not doing it quarterly, is more than the quarterly amount, whereas what we've seen is up to this point is all of these quarterly compounding interests have led to a higher output. So I want you to kind of think about that one. It's kind of an anomaly, but if you think through this, I think you'll see exactly why that's happening the way that it is.

So we'll move on from that. And as we get into this lesson and stuff, I'll talk some more about that, too, and why that's happening the way that it is.

So let's move on to the present value of uneven cash flows. And in this case here, we're going to use our NPV function to solve this problem. And this becomes very similar to the function that we use to solve for our capital budgeting exercise and problems. So it's important to learn this function and to know exactly how it works here. So we're going to demonstrate how this is working.

So in this case here, we have this series of cash flows, and we want to know what those cash flows are worth in today's dollars, in present value dollars. And they're going to be uneven cash flows across this five-year period. We see that we've got, in year one, $8,000, $4,500 in two and so forth all the way through. And they add up to $24,700.

Now we know from our time value of money axioms and analysis that in present value terms, that 24,700 is going to be something less than that because we're not receiving these payments until out there in the future. So let's do this calculation. Again, it's under Financials. It's called the NPV or Net Present Value function. It's really simple to use. It asks for the rate. And then it asks for these series of payments that are coming back to us. And we just drop these in individually.

We get down here to year 4. It looks like you run out of potential being able to add these. But you can see there is a drop down bar over here. You can add many, many values. And then you put in year five. And when we drop that in, we find out that this investment at 8% interest rate is worth $19,834 in today's dollars. We received $24,700, but in today's dollars it's worth $19,834.

Now there's something interesting about this. You can see here we changed up the sequence of these cash flows. We moved all of the larger cash flows to the beginning of this project. Even though they still add up to $24,700, we're going to calculate this, and I want you to see the difference between how the timing of these cash flows make a difference in our investment.

So let's run this one. And again, this is for net present value. It's the rate factor again. And then each of these cash flows, just like we did in this last exercise. And that last one dropped in. And what we find out is that this sequence of cash flows, in today's dollars, are worth $20,961. And it's important to notice that, again, even though they add up to the same total, we added all of those to the beginning, to year 1, the larger cash flows at the beginning of this project. And it's important when we make an investment decision, to note to know that, too, to determine when those cash flows come in and how the sequence of those and the importance of that. And this demonstrates that exact thing.

So you can see the difference between the $19,834 and what we got out of this the second one, this $20,961. So we'll use this net present value again for a handful of the problems that we're going to be solving. There's a couple things that we'll add to that, too, when we get into our capital budgeting to make sure that we account for the cash outflows that we have in those problems.

So let's move on to the rate calculation. In this case here, in this problem, this formula calculates the rate of return when the present value, future value, and the number of periods is known. So what we don't know here is we don't know what our rate of return is. We're trying to determine that. So we know the present value, which is $15,000 in this case. And we know the future value in this case, which is $28,000, and we know the number of periods. But we don't know what the return rate is for this investment.

But we have a calculation, or we have a formula, within Excel, again, that will be able to tell us that. And it's really easy to use. So let's go there and take a look at this. And so this formula, it's called Rate. And we'll use this calculation in our budgeting exercise, our budgeting and forecasting exercise. So it's important to learn how this actually works.

So in this case, the number of periods, we know, is five. There are no payments in this. But there is a present value, which is this $15,000. And we know that the future value that we'll be receiving is $28,000. And when we calculate this, we find that the rate of return that we're receiving here for this investment 13.296% a year. And that's compounded, which makes this-- it's the difficult part of this calculation.

And I'm going to show you how this works because we're going to use a manual formula to calculate this as well. So in this case here, we can write a manual formula that does the same thing. And I think it's important for you guys to see exactly what's going on here.

So in this calculation, we want to calculate what the rate of return is over this five-year period and with the compounding that is taking place over each period. So the way we start this calculation off is we start off with a 1. And it's 1 plus-- we want to calculate now the increase that has taken place from this $15,000 to 28. I convert this $15,000 over to a positive amount because it makes the formula easier. So basically here, I just multiply this negative $15,000 times minus 1.

So when we do this calculation, we're going to take this $28,000 where we ended up minus the $15,000 that we started with. And we divide that by our starting point or our present value. Now we have to get all of this under the same parentheses, balance our parentheses out.

Now I'm going to stop the formula here and we're going to take a look at what we have right now. And we need to convert this over to percent. And when we look at this, right now we're at 187%. Now that's not actually correct. The percentage increase from 15,000 to 28 is actually 87%. But we added this 1 into the calculation, and we need that because we're going to be raising this to a power. So that's why we did.

So this increase, actually, from $15,000 to $28,000 is actually an 87% increase. But that's over five periods. And we want to know what each of those individual periods are. And what complicates it is because they're each, then, compounding. So the way we do this is-- and we have to put another set of parentheses around this to encapsulate the 1 that we're working with here. Now we're going to raise this to a power again.

This is not real intuitive, though. We're going to raise it to a power. We want these individual years in between there. So we've been raising it to the power of the number of periods. And we're going to use that number of periods, but we're going to use it differently because really what we want is a fraction of that. So we're going to raise it to the 1/5 power. So it's 1 divided by the number of periods.

And when we do that-- well, let me back out of this formula right now and we can look at it. So when we do that, we still have that 1 in there that we have to subtract out. But we're at 113. So I'm pretty sure we're getting close here. So let's back that 1 out that we added into the formula. We need that when we raise it to a power. But we need to get rid of it when we're finishing the formula up. And then we'll take this out, and you can see we get the exact same formula that we calculated in Excel.

Now we can prove this. You may be saying, well, how do we know that that's actually correct? Well, we can go back to our future value formula that we've been using. And we're going to test this or prove this formula. And if we go to the future value and we take the rate that we just calculated, and the number of periods of five, and we take the present value of this $15,000, then we're able to generate this future value, which was 28. So we know that that is the correct interest rate that's taking place over each one of those five-year periods.

So that's our rate calculation. It's a powerful tool. We'll use it in a different way when we get into our statistics budget and when we're projecting from historical periods into the future. So it's a really powerful tool, and it's one that I want you guys to own, be able to put into your tool belt.

So let's move on to the investment calculator. And this was just kind of a fun exercise here. What we have is we have an investment. And if you go to *Forbes* or any of the financial magazines or websites and stuff, they have these financial calculators that you can use and stuff.

We're going to do the exact same thing here when we break it down into a couple different parts. So most people that have investments and whatever, you have kind of an initial investment. And then you have the amount that you're investing, say, in each period. So we're working with two different things that we just worked with in this exercise here. And that is a lump sum and then an annuity. So both of those coupled together, we can create this financial calculator.

So for this particular person we're going to say he has $250,000 in an initial investment. And then he's making yearly entries, or yearly payments to his investment of $18,000. We have the investment yearly return, which is 6%. And we have his years to retirement. So we're trying to calculate this as a retirement for this person.

So let's go through this. This first one, when we calculate his initial investment, is a lump sum. And we go back to our future value calculation, our rate, in this case, our investment yearly rate is 6%. But we're compounding this monthly, so we would divide it by 12. And then the number of periods are 15 years. But we have to multiply it times 12 because we're breaking this down into monthly payments, or 15 times 12 is the number of periods.

And then the present value of this is his starting point, which is this $250,000. Now we don't have that as a negative number. So we need to put a negative sign in this box and then call up the $250,000. When we do that, we find that this investment, in 15 years, his $250,000, has grown to $613,523. So that's his initial investment, what it's worth.

The second piece of this is the yearly amount that he's putting in. And we're going to calculate this one on a monthly basis as well. So it's going to be $18,000 divided by 12, and these other variables will be factored by 12 as well.

So in this one here, it's under future value as well. But we're going to use the payment piece of this. But the first thing it asks for is the rate again, which is 6% divided by 12. And the number of periods, , again, is over a 15-year period. So it's times 12. And then the payments here are going to be $18,000 divided by 12 because these are $18,000 worth of yearly payments. And we have to add that entry in as a negative as well in the payment piece to get a positive outlook.

So what we find is that over that 15-year period, If he makes payments of $18,000 a year, that those payments, the investment will be worth $436,000 at the end of 15 years. If we add both of those together, and then we put another factor on it for this particular person, we're able to generate an amount that he would be able to withdraw on a monthly basis. And we're saying 15. This person must have been 47 years old when they started this.

But this is a useful tool. It's a fun exercise. People like get in the class. We're not relating this part of it to business, although you certainly could. Businesses have investments like this that we can also put together and apply to a business. But you can use this for your own calculation and your own retirement and that type of thing as well. It's just a good tool.

What you're going to find out here is that if you kind of play with these different variables, there's one of these variables that trumps all of the rest. And I'll let you play with those. I'll give you a hint, though. In the time value of money, the number of years is extremely important. It's important for folks when they're investing in stuff, too. And that's why they always tell you to get started early and stuff, because these number of years play an incredible role.

If you just kind of play with those and see, if you doubled each of these and just held the other ones constant, you'll see that the years of retirement just has a huge impact on this, on the bottom line of these investments.

So that concludes this tutorial. And I'll see you guys in the next tutorial that we put together.