Design and Analysis of Algorithms

Problem #1.
A forensic lab receives a delivery of $n$ samples. They look identical, but in fact, some of them have different chemical composition. There is a device that can be applied to two samples and tells whether they are different or not. It is known in advance that most of the samples (more than 50%) are identical. Find one of the samples in the majority making no more than $n$ comparisons. (Beware: it is possible that two samples are identical but do not belong to the majority of identical samples.)

Problem #2.
Draw a single binary tree $T$ such that
a) each node of $T$ stores a single character from $\{A,E,F,M,N,U,X\}$,
b) a preorder traversal of $T$ yields EXAMFUN, and
c) a inorder traversal of $T$ yields MAFXUEN.

Problem #3.
Each of $n$ keys in an array may have one of the values red, white or blue. Give a linear time algorithm for rearranging the keys so that all the reds come before all the blues and all the blues come before all the whites. The only operations permitted on the keys are
- Examination of a key to find out what color it is.
- A swap (interchange of positions) of two keys specified by their indices.